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Symmetric Missile Dynamic Instabilities

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Nomenclature

a $= \left(\frac{\text{cubic planar damping moment}}{\text{cubic circular damping moment}}\right) - 1$
b_1 $= [4(s_g - 1)C^*_{M_\alpha}]^{1/2}$
b_2 $= -b_1$
C_l, C_m, C_n $= XYZ$ components of $\left(\frac{\text{aerodynamic moment}}{\frac{1}{2}\rho S l V^2}\right)$
$C_{\tilde{m}}, C_{\tilde{n}}$ $= \tilde{Y}\tilde{Z}$ components of $\left(\frac{\text{aerodynamic moment}}{\frac{1}{2}\rho S l V^2}\right)$
C_{M_d} = damping moment part of $C_m + iC_n$
C_{M_α} $= (\frac{1}{2})(C_{m_\alpha} - C_{n_\beta})$
$\hat{C}_{M_\alpha}$ $= (\frac{1}{2})(C_{m_\alpha} + C_{n_\beta})$
C_{SM_α} $= \left(\frac{\text{induced side moment}}{\frac{1}{2}\rho S l V^2 \delta}\right)$
$\hat{c}$ $= \left(\frac{\text{Magnus moment}}{\frac{1}{2}\rho S l V^2 p \delta}\right)$
I_x, I_y = axial and transverse moments of inertia
K_1, K_2 = amplitudes of the two yaw model arms for a symmetric missile
k_3 = trim angle magnification factor
l = reference length
m = mass
p,q,r $= XYZ$ components of the missile's angular velocity
S = reference area
s_d = dynamic stability factor $= -2\hat{c}_0/d_0\sigma$
s_g = gyroscopic stability factor $= (\sigma p)^2/4C^*_{M_\alpha}$
t = time
V = magnitude of the missile velocity
XYZ = missile-fixed axes, the X axis along the longitudinal axis of the missile, positive forward
$X\tilde{Y}\tilde{Z}$ = nonspinning aeroballistic axes
α, β = angles of attack and sideslip in the missile-fixed system
δ $= |\xi|$
δ_c = circular limit motion radius
θ = polar angle of ξ
λ_j $= \dot{K}_j/K_j$ $(j = 1,2)$
λ_{j0} = value of λ_j for a linear Magnus moment
ξ = the complex angle of attack in the missile-fixed system $= \beta + i\alpha = \delta e^{i\theta}$
$\tilde{\xi}$ $= \beta - i\alpha$
ξ_a = trim angle $= (C_{m_0} + iC_{n_0})/iC_{M_\alpha}$
ρ = air density
σ $= I_x/I_y$
ϕ_α = zero-spin pitch frequency
ϕ_β = zero-spin yaw frequency
ϕ_j = orientation angle of the jth yaw modal arm $(j = 1,2,4,5)$

Superscripts

$(\dot{\ })$ $= \mathrm{d}(\ )/\mathrm{d}t$
$(\ )^*$ $= (\ ) \times (\rho S l V^2/2I_y)$
$(\tilde{\ })$ = nonspinning aeroballistic system version of ( )

Note: the real constants C_{l_0}, C_{l_p}, C_{m_0}, C_{m_q}, C_{m_α}, $C_{m_{\dot{\alpha}}}$, C_{n_0}, C_{n_r}, C_{n_β}, $C_{n_{\dot{\beta}}}$, c_0, c_2, $\hat{c}_0$, $\hat{c}_2$, d_0, d_2, K_{1R}, K_3, K_4, K_{4R}, and K_5 having meanings that are clear from the equations in which they first appear.

Introduction

PROBABLY the first contribution of missile stability analysis was the placement of feathers on the arrow. This created a restoring moment and made the arrow a statically stable missile whose angular motion is approximated by a sine wave. Early bullet designers soon realized that feathers or fins are most inconvenient for gun launch and found it necessary to impart a high spin rate to the bullet or shell by means of rifled tubes. If sufficient spin is given the projectile, it is gyroscopically stable and its angular motion is approximated by the sum of two sine waves. From a dynamics point of view, the motion of a symmetric statically stable missile is a special case of the motion of a gyroscopically stable missile.

Until World War I, the primary concern of the designer was static or gyroscopic stability and the most important moments were the static moments, which were assumed to be linear in

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the angles of attack and sideslip:

$$C_m = C_{m_\alpha}\alpha + C_{m_0} \tag{1}$$

$$C_n = C_{n_\beta}\beta + C_{n_0} \tag{2}$$

The two transverse static moment terms can be combined into a convenient complex variable form

$$C_m + iC_n = -iC_{M_\alpha}(\xi - \xi_a) + i\hat{C}_{M_\alpha}\bar{\xi} \tag{3}$$

where

$$\xi = \beta + i\alpha$$

If $\hat{C}_{M_\alpha}$ is not zero, the aerodynamic moment of Eq. (3) is essentially asymmetric and the missile's angular motion has the complexity of aircraft motion. If both $\hat{C}_{M_\alpha}$ and ξ_a are zero, the aerodynamic moment is symmetric and the resulting angular motion is that of a body of revolution. For the intermediate case of zero $\hat{C}_{M_\alpha}$ and nonzero ξ_a, the missile has a *slight* aerodynamic asymmetry and its moment is that of a body of revolution with respect to the complex aerodynamic trim angle ξ_a.

A symmetric missile is statically stable if C_{M_α} is negative; this is the usual design requirement for finned missiles. Artillery projectiles have positive C_{m_α}'s and are required to have high spin rates to achieve gyroscopic stability. A fairly complete linear theory of the pitching and yawing motion of spinning projectiles was first published[1] by English exterior ballisticians in 1920 and was refined by McShane et al.[2] during and following World War II.

Since World War II missile designers have encountered a number of surprises with regard to the dynamic stability of their designs. The dynamic stability is influenced by additional moment terms and determines the growth or decay of the oscillatory angular motion. In this paper, we will give a survey of dynamic instabilities that have been observed as well as some that are possible but not yet observed. For most of this paper, only symmetric missiles or missiles with slight asymmetries will be considered. A brief discussion of an almost symmetric missile ($|\hat{C}_{M_\alpha}| \ll |C_{M_\alpha}|$) as well as the effect of moving payloads will also be given. Equal transverse moments of inertia will be assumed throughout the paper and the small effect of lift and drag forces on the angular motion will be neglected.

Nonspinning Symmetric Missiles

In addition to the static moment coefficients, the linear motion of a nonspinning missile is affected by four damping moment coefficients: $C_{m_q}, C_{n_r}, C_{m_{\dot\alpha}}$, and $C_{n_{\dot\beta}}$. For the oscillatory motion, the angular derivatives are related.

$$q \doteq \dot\alpha \qquad r \doteq -\dot\beta \tag{4}$$

The symmetry assumption, then, allows the following simple expression for the complete linear transverse moment:

$$C_m + iC_n = -i(C_{M_\alpha}\xi + d_0\dot\xi) \tag{5}$$

where

$$d_0 = (l/V)(C_{m_q} + C_{m_{\dot\alpha}}) = (l/V)(C_{n_r} - C_{n_{\dot\beta}})$$

The general angular motion of a spinning symmetric missile with a linear aerodynamic moment is two-mode epicyclic motion like that shown in Fig. 1. Mathematically it is generated by the sum of two two-dimensional vectors rotating at constant rates with amplitudes K_j that damp exponentially. The sum of these modal amplitudes is the maximum value of the total angle of attack while their difference is the minimum value of this angle. In complex notation this can be expressed as

$$\xi = K_1 e^{i\phi_1} + K_2 e^{i\phi_2} \tag{6}$$

$$\dot K_j = \lambda_j K_j \tag{7}$$

For the special case of a nonspinning statically stable missile, the damping rates are equal and the angular rates are equal in magnitude and opposite in sign

$$\lambda_1 = \lambda_2 = \tfrac{1}{2} d_0^* \tag{8}$$

$$\dot\phi_1 = -\dot\phi_2 = \sqrt{-C^*_{M_\alpha}} \tag{9}$$

The motion is a damped ellipse with semimajor axis $K_1 + K_2$ and semiminor axis $|K_1 - K_2|$. It is important to note that $\lambda_1 = \lambda_2$ implies that the eccentricity of the elliptical motion is maintained. Thus, initially planar motion remains planar and initially circular motion remains circular.

According to the definition of λ_j, the motion will be unstable if d_0^* is positive and the aerodynamic damping moment acts to increase the angular rates. This unexpected behavior has been observed at hypersonic speeds for slowly spinning reentry shapes. This dynamic instability could be caused by the entropy gradient induced by the bow shock and reinforced by ablation.[3] At transonic speeds, unstable damping has been observed by MacAllister[4] and this has been explained by nose-induced separation.[5]

An important feature of MacAllister's ballistic range measurements was that the initial almost-planar motion quickly became an oval almost-circular limit motion. The theoretical explanation of this limit motion requires the consideration of in-plane and out-of-plane damping. An "in-plane moment" means a moment producing a rotation in the plane of the total angle of attack; the in-plane moment vector is thus normal to this plane. Similar remarks apply to the out-of-plane moment. If we rewrite the linear damping moment terms of Eq. (5) using the polar form of the angle of attack, $\xi = \delta e^{i\theta}$, we have

$$C_{M_d} = -id_0(\dot\delta + i\dot\theta\delta)e^{i\theta} \tag{10}$$

Thus for linear theory, the damping moment in the plane of the total angle of attack is proportional to $\dot\delta$, the radial rate of change of this angle; the damping moment perpendicular to this plane is proportional to $\dot\theta\delta$, the circumferential rate of change of this angle; and the proportionality factors are equal. These in-plane and out-of-plane moments are shown in Fig. 2.

Since the symmetry assumption requires that the damping moment be an odd function of the angle, the simplest

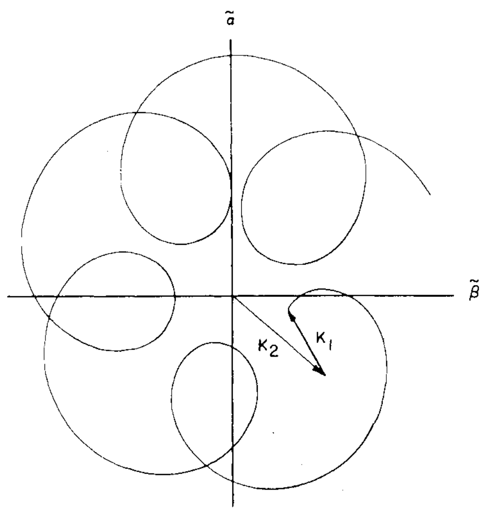

Fig. 1 Typical epicyclic motion of a spin-stabilized shell.

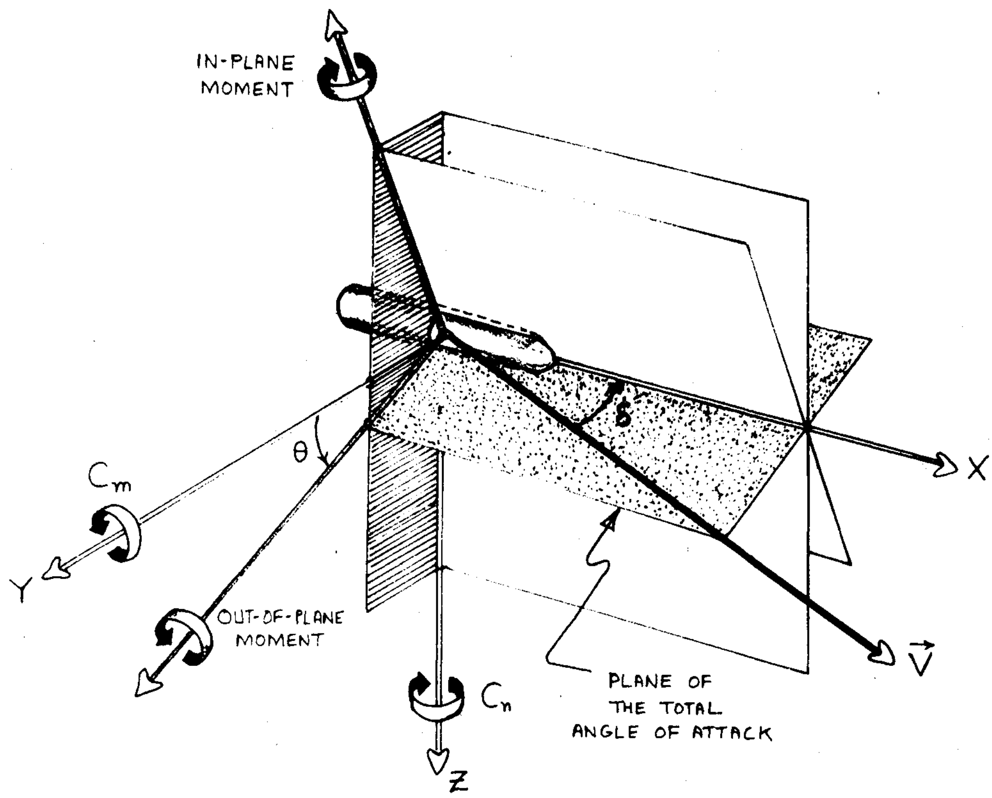

Fig. 2 In-plane and out-of-plane moments.

nonlinear expression for the damping moment is

$$C_{M_d} = -i(d_0+d_2\delta^2)(\dot{\delta}+i\dot{\theta}\delta)e^{i\theta} \tag{11}$$

A more general expression that retains the in-plane and out-of-plane damping equality is obtained by replacing the constant d_2 by a function of δ^2. Even this assumption is not sufficient to generate the circular limit motion observed by MacAllister.

A successful approach[6] is to drop the equality of in-plane and out-of-plane damping:

$$C_{M_d} = -i\{d_0(\dot{\delta}+i\dot{\theta}\delta)+d_2\delta^2[(1+a)\dot{\delta}+i\dot{\theta}\delta]\}e^{i\theta}$$

$$= -i[(d_0+d_2\delta^2)\dot{\xi}+d_2a\delta\dot{\delta}\xi] \tag{12}$$

For constant d_2 and a, Eq. (12) introduces two cubic damping terms.[7] This nonlinear moment expression can be used in the usual quasilinear analysis.[8-11] According to this analysis, the nonlinear solution can be approximated by a solution of the form of Eqs. (6) and (7) in which the λ_j's become functions of the K_j's.

$$\lambda_1 = ½[d_0^*+d_2^*(K_1^2+aK_2^2)] \tag{13}$$

$$\lambda_2 = ½[d_0^*+d_2^*(K_2^2+aK_1^2)] \tag{14}$$

The behavior of a nonlinear solution can be described by trajectories in a K_1^2 vs K_2^2 amplitude plane. Since Eq. (6) for $\lambda_j=0$ generates ellipses, each point in the amplitude plane identifies an elliptical motion and the trajectory through that point describes how this elliptical motion changes under the influence of nonlinear damping. Points on the amplitude plane axes represent circular motions and the line $K_1^2=K_2^2$ is the locus of planar motions.

If d_0 and d_2 are opposite in sign, a circular singularity exists with amplitude $\delta_c=(-d_0/d_2)^{½}$. The amplitude plane for this case and for equal in-plane and out-of-plane damping ($a=0$) is given in Fig. 3. The circular limit motions are unstable but there is a stable planar limit motion with amplitude $2\delta_c$. It can be shown that for $a<1$ the circular motions are unstable but for $a>1$ they are stable. Figure 4 gives the amplitude plane for $a=2$. Numerical integrations of the complete equations of motion verify that stable circular limit motions do exist for $a>1$. Recently, several authors[12-15] have made a variety of wind tunnel measurements of out-of-plane damping and have shown that it can be quite different from in-plane damping for cones at supersonic and hypersonic speeds. Since $1+a$ is the ratio of the planar damping to the circular damping, we see that this ratio must exceed two before stable circular motion can exist.

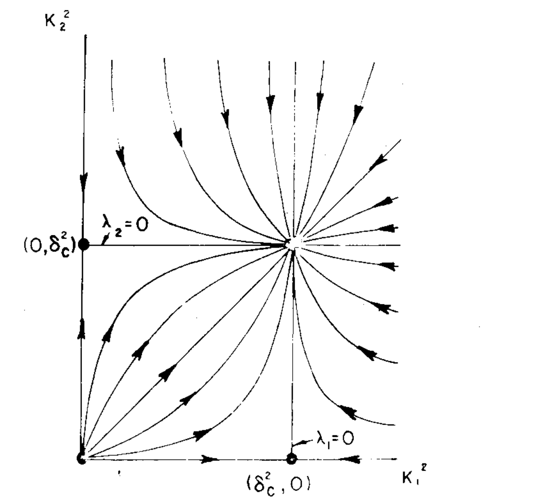

Fig. 3 Amplitude plane for equal in-plane and out-of-plane damping moments.

One common feature of Figs. 3 and 4 is that large-amplitude planar motions decay and small-amplitude motions grow. Although these motions go to different limit motions, the final motions are bounded. Thus, we could expect that for all cases when planar damping-in-pitch wind tunnel measurements show similar behavior, *bounded* flight motions would occur. Figure 5, for d_0 and d_2 both positive and $a<-1$, shows that all nonplanar motions go to large spiral

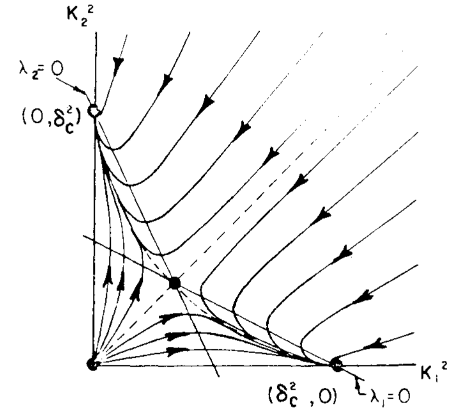

Fig. 4 Amplitude plane for an in-plane/out-of-plane cubic damping moment ratio of three.

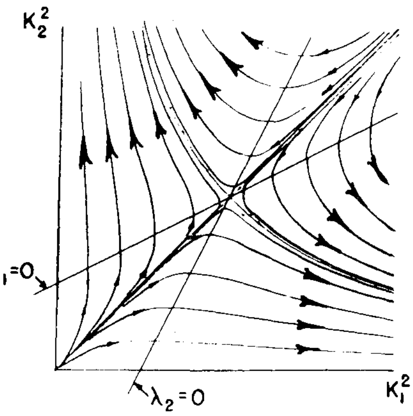

Fig. 5 Amplitude plane for a negative in-plane/out-of-plane cubic damping moment ratio.

motions although planar motions tend to the planar singular motion with amplitude $2[-(1+a)d_2/d_0]^{-1/2}$. This shows that intuitive arguments should be applied with care to nonlinear systems.[16]

Linear Magnus Moment

For a statically unstable missile such as a shell or bullet, high rates of spin are required for stability and a Magnus side moment must be added to the total aerodynamic moment. Since statically stable missiles are usually spun to reduce the effect of manufacturing asymmetries, this Magnus moment should be considered for finned missiles as well as bodies of revolution.

Before doing so, we must make a decision on the appropriate coordinate system. In order to avoid the algebraic complexities of a spinning missile-fixed XYZ axis system, we will use nonspinning aeroballistic coordinates $X\tilde{Y}\tilde{Z}$. The X axis pitches and yaws with the missile. The $\tilde{Y}$ axis is selected to lie in the horizontal plane initially but it is very important to note that it does not remain there. If this axis were required to remain in the horizontal plane, the spin of the system would be nonzero and these axes would be called fixed-plane axes. Fixed-plane axes are useful when horizontal or vertical forces such as gravity are present.[17] Indeed, it has been shown that an ascending or descending missile with a constant horizontal control moment could have an instability due to the terms involving fixed-plane coordinate spin.[18-19]

In aeroballistic coordinates, the complex angle of attack is written as $\tilde{\xi}$ and the transverse aerodynamic moment coefficients become

$$C_{\tilde{m}} + iC_{\tilde{n}} = (p\hat{c} - iC_{M_\alpha})\tilde{\xi} - id_0\dot{\tilde{\xi}} \tag{15}$$

The first term on the right side of Eq. (15) represents the Magnus moment, which for constant $\hat{c}$ is proportional to the spin and the angle of attack. A positive Magnus moment coefficient represents a moment that acts to rotate the nose of the missile around the velocity vector in the direction of spin.

The presence of spin and linear Magnus moment ($\hat{c}=\hat{c}_0$) does not change the form of the epicyclic solution of Eq. (6) but does give more complicated damping rates and frequencies.

$$\lambda_j = \lambda_{j0} \equiv (\dot{\phi}_j d_0^* + p\hat{c}_0^*)/b_j \qquad (j=1,2) \tag{16}$$

$$\dot{\phi}_j = \tfrac{1}{2}(\sigma p + b_j) \qquad (j=1,2) \tag{17}$$

Dynamic stability requires negative λ_j's and can be stated simply in terms of two stability factors, s_g and s_d, where

$$s_g = (\sigma p)^2/4C^*_{M_\alpha} \tag{18}$$

$$s_d = -2\hat{c}_0/d_0\sigma \tag{19}$$

The gyroscopic stability factor s_g is essentially the ratio of squared gyroscopic spin to static moment coefficient. Periodic motion occurs when this stability factor is greater than unity. The dynamic stability factor s_d is essentially the ratio of the Magnus moment coefficient to the damping moment coefficient. For dynamic stability[20]

$$1/s_g = 4C^*_{M_\alpha}/(\sigma p)^2 < (2-s_d)s_d \tag{20}$$

Figure 6 summarizes all the implications of this relation. Note that if s_d is outside the interval (0,2), a statically unstable missile cannot be dynamically stabilized by spin and a statically stable missile can be made dynamically *unstable* by sufficiently high rates of spin. Very simple cone cylinders and finned cone cylinders have been shown to have s_d values outside this region.[21] The linear Magnus moment is largest for long projectiles with boattails at transonic speeds. Indeed, for an early version of the M483 shell,[22] the linear Magnus moment had a transonic peak value which placed s_d well outside this region. This Magnus instability produced a number of quite dramatic 3 km shorts when the shell was fired at Mach numbers between 0.92 and 0.96. This undesirable behavior was eliminated by shortening the boattail by 50% and thereby significantly reducing the linear Magnus moment.

Nonlinear Magnus Moment

In 1951, a very strange problem[23,24] arose during the development of the 12.75 in. antisubmarine ship-launched spinning finned rocket, the Weapon A. When fired to the port side of a high-speed destroyer, it performed well. When fired to the starboard side, however, its angular motion grew to a very large amplitude coning motion and its performance was completely unsatisfactory.

If a missile's stability depends on launch conditions, the differential equation characterizing the motion must be nonlinear. In the case of Weapon A, the cause was a strongly nonlinear Magnus moment. The behavior can be easily predicted by the quasilinear theory for a simple cubic Magnus moment, i.e., a quadradic Magnus moment coefficient:

$$\hat{c} = \hat{c}_0 + \hat{c}_2\delta^2 \tag{21}$$

For this moment, the quasilinear exponential damping functions that determine trajectories in the amplitude plane become

$$\lambda_1 = \lambda_{10} + p\hat{c}_2^*(K_1^2 + 2K_2^2)/b_1 \tag{22}$$

$$\lambda_2 = \lambda_{20} + p\hat{c}_2^*(K_2^2 + 2K_1^2)/b_2 \tag{23}$$

A possible amplitude plane[8] for these λ_j's is given in Fig. 7. According to Eq. (17), the frequencies for a finned projectile ($C_{M_\alpha} < 0$) are opposite in sign and thus its complex angular

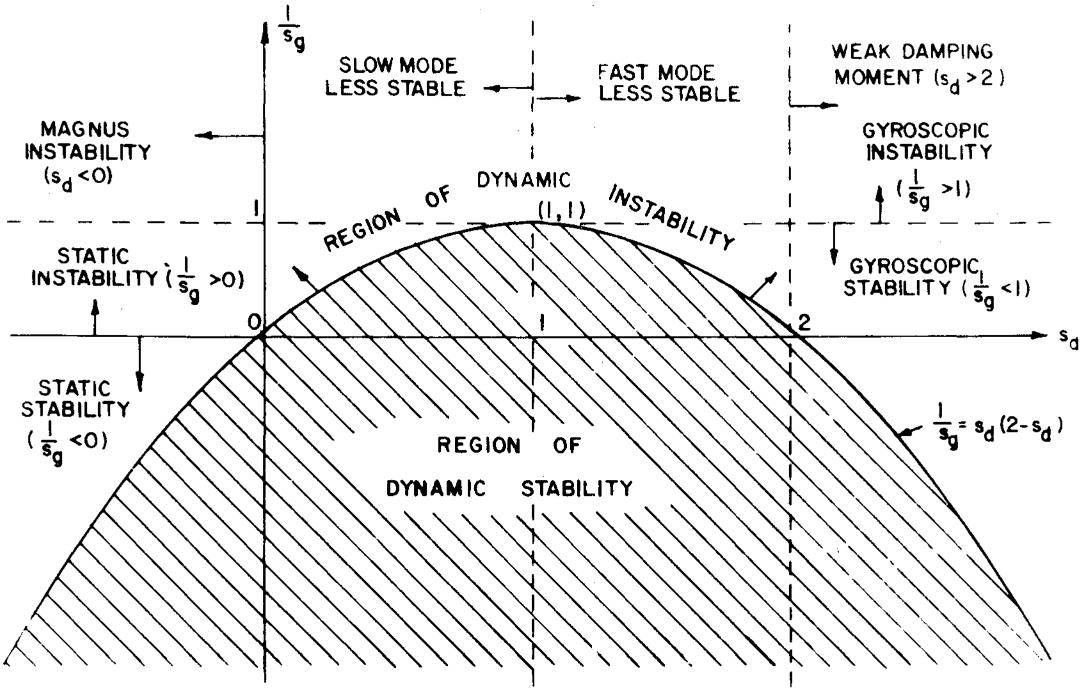

Fig. 6 Stability regions in the $s_d - 1/s_g$ plane.

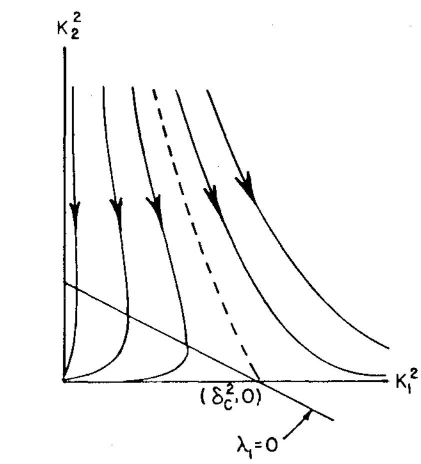

Fig. 7 Amplitude plane for a cubic Magnus moment.

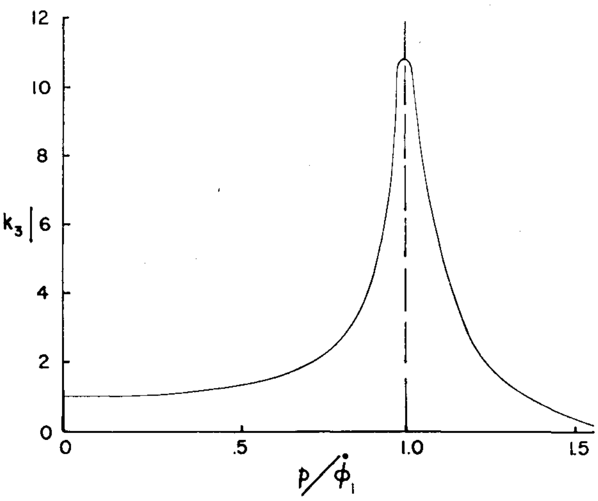

Fig. 8 Trim magnification factor amplitude (for constant spin) vs spin ratio.

motion is described by the sum of a K_1 vector rotating counterclockwise in the direction of spin and a clockwise rotating K_2 vector. Gravity tipoff plus the crosswind produced by launch to the port of ships moving at 30 knots produces 10 deg amplitude clockwise angular motion. Launch to starboard produces 10 deg amplitude counterclockwise motion. If $\delta_c = 5$ deg, the motion associated with port launch lies to the left of the dashed curve (the separatrix) and will damp to a small-amplitude coning motion. Starboard launch is to the right of the separatrix and large-amplitude motion is successfully predicted by the theory.

Aerodynamic Trim ($\xi_a \neq 0$)

The aerodynamic moment of a symmetric missile will not be zero at zero angle of attack if its body or fins are slightly deformed or its center of mass is not on its axis of symmetry. This trim moment causes the ξ_a in Eq. (3). In aeroballistic coordinates, the resulting trim angle rotates with the missile and has an amplitude that depends on the spin rate.[25,26]

$$\tilde{\xi} = K_1 e^{i\phi_1} + K_2 e^{i\phi_2} + \xi_a k_3 e^{ipt} \tag{24}$$

$$k_3 \doteq \dot\phi_1 \dot\phi_2 (p - \dot\phi_2)^{-1} (p - \dot\phi_1 + i\lambda_1)^{-1} \tag{25}$$

For most spins, λ_1 can be neglected. Near resonance, $p = \dot\phi_1$, it is important and determines the maximum value of k_3. A plot of $|k_3|$ vs p is given in Fig. 8 and shows the need for missile designers to avoid resonance.

Induced Roll Moment ($\xi_a \neq 0$)

In general, roll motions are such that a missile's roll rate will vary through resonant spin rate and the missile will attain only a fraction of its maximum trim value.[27,28] If the roll moment at angle of attack is a function of θ, the roll orientation of the plane of the angle of attack, the roll moment coefficient can be written in the form[29]

$$C_l = C_{l_0} + (pl/V) C_{l_p} + \delta C_{l_\delta}(\theta, \delta) \tag{26}$$

$\tilde{\xi}$ in Eq. (24) can be expressed in terms of θ and δ and the result used to compute these variables.

$$\delta e^{i(pt+\theta)} = K_1 e^{i\phi_1} + K_2 e^{i\phi_2} + \xi_a k_3 e^{ipt} \tag{27}$$

For most motions, θ varies rapidly and the rolling motion is unaffected by C_{l_δ}. For pure trim motion ($K_1 = K_2 = 0$) or two-mode motion near resnonance ($K_2 = 0$, $\dot{\phi}_1 \doteq p$), θ is constant and C_{l_δ} can have an important effect. Near resonance, δ can be quite large and the resulting induced roll moment [i.e., the third term in Eq. (26)] can force the roll rate to stay near resonance. This phenomenon of "rock lock-in" has been observed in flight as well as in a number of computer simulations.

The induced roll moment can be caused by aerodynamic asymmetry present in a "symmetric" four-finned missile or by mass asymmetries in an aerodynamically symmetric missile. For example, the lift force on a symmetric missile at angle of attack acts on the center of mass of the missile. If the center of mass is not on the missile's axis of symmetry, it will produce a roll moment of the form of Eq. (26). It should be emphasized, however, that the occurrence of roll lock-in depends on the details of the pitching motion and its coupling to the rolling motion through Eq. (26) and can only be determined by numerical integrations.[30-32]

Induced Side Moment ($\xi_a \neq 0$)

Although flight failures have been explained by the occurrence of resonance through roll lock-in, in some cases angles of attack have been observed much larger than those predicted by Eq. (25). In 1959, Nicolaides[29] developed his "catastrophic yaw" theory by the introduction of induced side moments. The existence and the effect of these moments have been discussed by other authors.[33-35] Nicolaides' induced side moment term can be included in the aerodynamic moment expression of Eq. (15) by adding $[C_{SM_\alpha}(\theta,\delta)]\tilde{\xi}$ to the right side of Eq. (15).

For flight conditions for which θ is constant, the primary effect of this term is to change λ_1 to

$$\lambda_1 = \lambda_{10} - C^*_{SM_\alpha} b_1^{-1} \tag{28}$$

The presence of the induced side moment coefficient in Eq. (28) introduces the possibility of a very small λ_1, which can cause a very large resonance value of k_3 in Eq. (25). An even worse possibility is a large positive value of λ_1, which would cause an exponential growth of K_1. This possibility is the catastrophic yaw of Nicolaides and may be the cause of some spectacular flight failures.

Nonlinear Aerodynamic Moment ($\xi_a \neq 0$)

The combination of nonlinear aerodynamic moments with a trim moment can give rise to a rich variety of limit motions. Many of these motions have been produced by computer simulations but as yet none has been observed in flight. In this section, we will briefly consider the effect of 1) a cubic static moment, and 2) two cubic damping moments.

First, the static moment coefficient is assumed to have the form

$$C_m + iC_n = -i[(c_0 + c_2\delta^2)\xi - c_0\xi_a] \tag{29}$$

For pure trim motion, this assumption replaces the linear Eq. (25) by a cubic equation for k_3. Near resonance, three values of k_3 can be computed but only two correspond to stable motion. In addition to these resonance motions, much more complicated limit motions have been found which are generalized subharmonic motions. For these motions, certain constant values of K_1, K_2, k_3 have been predicted and produced by computer integrations for spins far from resonance as well as quite near to resonance.[36,37]

A second set of limit motions can be constructed for the damping moment expansion of Eq. (12):

$$C_m + iC_n = -iC_{M_\alpha}(\xi - \xi_a) + C_{M_d} \tag{30}$$

A number of one-, two-, and three-mode limit motions have been predicted by the quasilinear theory and computed by numerical integration of the equations of motion.[38]

Moving Internal Parts

In 1955, an 8-in. shell, the T317, showed a strange spin decay coupled with a significant range loss.[39] In Fig. 9, the spin histories of three of these shells are compared with the spin histories of four T347s. The T347 had the same aerodynamic shape and mass distribution as the T317. As can be seen from the figure, the T317 had range losses of between 1% and 11% and associated spin losses. The range loss was due to a growth in the high-frequency component of the pitching motion ($\lambda_1 > 0$). A quite similar behavior was observed in recent ballistic range tests of a 20 mm projectile with the M505 fues which showed a growth of the high-frequency mode and an unexplained spin decay.[40] Both the T317 projectile and the M505 fuse carried components that could move a small amount during flight.

These phenomena have been explained[41,42] by assuming either 1) a forced circular motion of an internal part about the axis of the projectile (hula-hoop motion) or 2) a forced precession of the spin axis of the internal part about the spin axis of the projectile. In both cases, only the Fourier component of the motion at the higher coning frequency of the projectile was considered. Thus, a resonance was considered for which the amplitude of the internal motion was constant.

The theory predicted a relationship between the growth of the higher frequency mode and the spin decay and in both cases excellent quantitative agreement was obtained. Therefore, the derived expressions can be used to set tolerances for manufacture of these projectiles. Recently an extension of this theory was used to predict the adverse effect of an interior cantilever beam.[43]

Almost Symmetric Missiles

Throughout this paper we have assumed that $\hat{C}_{M_\alpha}$ in Eq. (3) was zero and neglected the essential asymmetry present when the pitch and yaw frequencies are not equal for a nonspinning missile. The influence of this airplane-like asymmetry on stability of a spinning vehicle was studied by Phillips[44] in 1948 and developed further in two recent very elegant papers by Hodapp.[45,46] The concept of an almost symmetric missile[47] was introduced in 1978 to show how a "little bit" of this essential asymmetry would affect the classical tricyclic theory of a symmetric missile with aerodynamic trim.

If $\hat{C}_{M_\alpha}$ is not zero, the pitch frequency $\dot{\phi}_\alpha$ of a nonspinning missile is not equal to its yaw frequency $\dot{\phi}_\beta$. If the spin rate is outside the region between these two frequencies (the resonance region), the motion can be described by a pentacycle:

$$\tilde{\xi} = \sum_{j=1}^{5} K_j e^{i\phi_5} \tag{31}$$

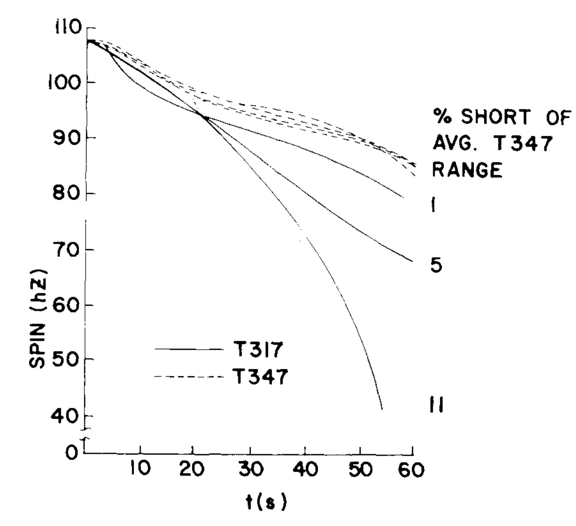

Fig. 9 Measured spin histories for the T317 and T347 shell.

where

$$\phi_3 = pt + \phi_{30} \qquad \dot{\phi}_4 = 2p - \dot{\phi}_1 \qquad \dot{\phi}_5 = 2p - \dot{\phi}_2$$

$$\left.\begin{array}{l} K_4/K_1 \to 0 \\ K_5/K_2 \to 0 \end{array}\right\} \qquad \hat{C}_{M_\alpha}/C_{M_\alpha} \to 0$$

When the spin is in the resonance region,

$$\tilde{\xi} = [K_{1R} e^{\lambda t + i\theta_R} + K_{4R} e^{-(\lambda t + i\theta_R)} + K_3] e^{i\phi} + K_2 e^{i\phi_2} + K_5 e^{i\phi_5} \tag{32}$$

According to Eq. (32), the motion grows exponentially when the spin is in the resonance region. Another unpleasant characteristic is the existence of peak values of K_3 at the endpoints of the region ($\dot{\phi} = \dot{\phi}_\alpha$ and $\dot{\phi} = \dot{\phi}_\beta$).

Four specific characteristics of the motion of almost symmetric missiles ($|\hat{C}_{M_\alpha}| \ll |C_{M_\alpha}|$) are as follows.

1) The general motion is well approximated by a symmetric missile with average coefficients.

2) Far from zero spin or resonance spin rates, the first observable modification of the usual tricyclic motion for an almost-symmetric missile is the appearance of a $2\dot{\phi} - \dot{\phi}_1$ frequency, followed by the appearance of a $2\dot{\phi} - \dot{\phi}_2$ frequency as the asymmetry becomes greater.

3) Near zero spin, both of these additional frequencies have substantial amplitudes, and near resonance, the $2\dot{\phi} - \dot{\phi}_1$ frequency has a substantial amplitude.

4) For spin in the resonance region, large trims and exponential undamping are possible.

Summary

As can be seen from the examples of this survey, a number of causes of dynamic instability exist for basically symmetric missiles. These include: 1) unstable linear damping moment, 2) nonlinear and unequal in-plane and out-of-plane damping moments, 3) linear and nonlinear Magnus moments, 4) spin-yaw resonance of missiles with trim, 5) spin lock-in and induced side moment acting on missiles with trim, 6) nonlinear damping moments acting on missiles with trim, 7) moving internal components, 8) spin in resonance region of "almost" symmetric missiles, and 9) none of the preceding.

Item 9 is naturally of constant concern to a designer but is clearly difficult to discuss. One area which should be considered is that of payload interactions. For example, very complicated instabilities have been encountered which are related to liquid payloads and solid-liquid mixed payloads. For this reason, the most active new work in symmetric missile stability is concentrated on these unusual payloads. Some parts of this work are extensions of World War II studies[48-52] but parts are new experimental and theoretical approaches that appear to be very promising.[53-55]

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